The Thermal Resistance Concept

The Fourier equation, for steady conduction through a constant area plane wall, can be written:

$$\dot{Q}_{Cond} = -kA\frac{dT}{dx} = kA\frac{T_1 - T_2}{L}$$

This can be re-arranged as:

$$\dot{Q}_{Cond} = \frac{T_2 - T_1}{R_{wall}} \qquad (W)$$

$$R_{wall} = \frac{L}{kA} \qquad (^{\circ}C/W)$$

R_{wall} is the *thermal resistance* of the wall against heat conduction or simply the *conduction resistance* of the wall.

The heat transfer across the fluid/solid interface is based on *Newton's law of cooling:*

$$\dot{Q} = hA(T_s - T_\infty) \qquad (W)$$

$$R_{Conv} = \frac{1}{hA} \qquad (^{\circ}C/W)$$

R_{conv} is the thermal resistance of the surface against heat convection or simply the *convection resistance* of the surface.

Thermal radiation between a surface of area *A* at T_s and the surroundings at T_∞ can be expressed as:

$$\dot{Q}_{rad} = \varepsilon\sigma A\left(T_s^4 - T_\infty^4\right) = h_{rad}A(T_s - T_\infty) = \frac{T_s - T_\infty}{R_{rad}} \qquad (W)$$

$$R_{rad} = \frac{1}{h_{rad}A}$$

$$h_{rad} = \varepsilon\sigma\left(T_s^2 + T_\infty^2\right)(T_s + T_\infty) \qquad \left(\frac{W}{m^2K}\right)$$

where $\sigma = 5.67\times10^{-8}$ [W/m^2K^4] is the Stefan-Boltzman constant. Also $0 < \varepsilon < 1$ is the emissivity of the surface. Note that both the temperatures must be in Kelvin.

Thermal Resistance Network

Consider steady, one-dimensional heat flow through two plane walls in series which are exposed to convection on both sides, see Fig. 2. Under steady state condition:

rate of heat convection into the wall	=	rate of heat conduction through wall 1	=	rate of heat conduction through wall 2	=	rate of heat convection from the wall

$$Q^{\bullet} = h_1 A\left(T_{\infty,1} - T_1\right) = k_1 A \frac{T_1 - T_2}{L_1} = k_2 A \frac{T_2 - T_3}{L_2} = h_2 A\left(T_2 - T_{\infty,2}\right)$$

$$Q^{\bullet} = \frac{T_{\infty,1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/k_1 A} = \frac{T_2 - T_3}{L/k_2 A} = \frac{T_2 - T_{\infty,2}}{1/h_2 A}$$

$$Q^{\bullet} = \frac{T_{\infty,1} - T_1}{R_{conv,1}} = \frac{T_1 - T_2}{R_{wall,1}} = \frac{T_2 - T_3}{R_{wall,2}} = \frac{T_3 - T_{\infty,2}}{R_{conv,2}}$$

$$Q^{\bullet} = \frac{T_{\infty,1} - T_{\infty,2}}{R_{total}}$$

$$R_{total} = R_{conv,1} + R_{wall,1} + R_{wall,2} + R_{conv,2}$$

Note that A is constant area for a plane wall. Also note that the thermal resistances are in series and equivalent resistance is determined by simply adding thermal resistances.

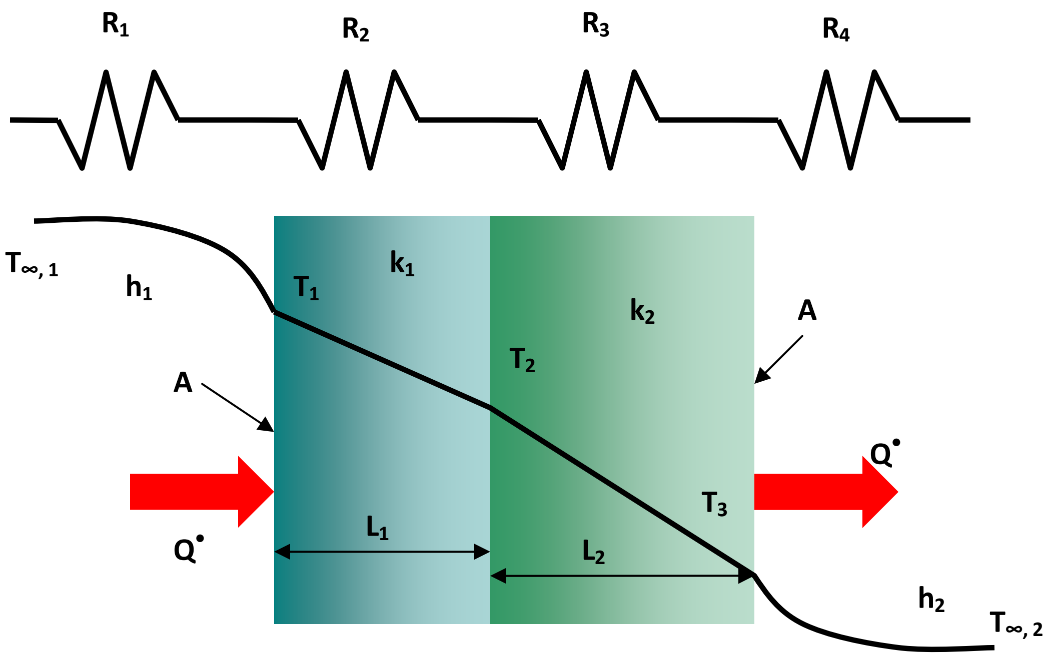


Fig. 2: Thermal resistance network.

The rate of heat transfer between two surfaces is equal to the temperature difference divided by the total thermal resistance between two surfaces.

It can be written:

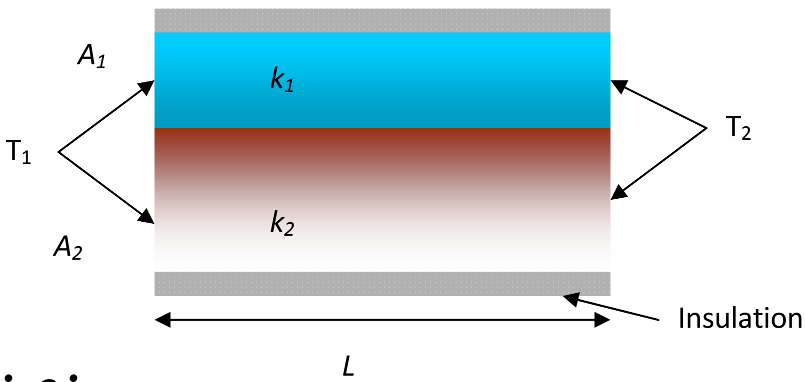
$$\Delta T = Q^{\bullet} R$$

The thermal resistance concept is widely used in practice; however, its use is limited to systems through which the rate of heat transfer remains constant. It other words, to systems involving *steady* heat transfer with no *heat generation.*

Thermal Resistances in Parallel

The thermal resistance concept can be used to solve steady state heat transfer problem in parallel layers or combined series-parallel arrangements.

It should be noted that these problems are often two- or three dimensional, but approximate solutions can be obtained by assuming one dimensional heat transfer (using thermal resistance network).



$Q^{\bullet} = Q_2^{\bullet} + Q_2^{\bullet}$

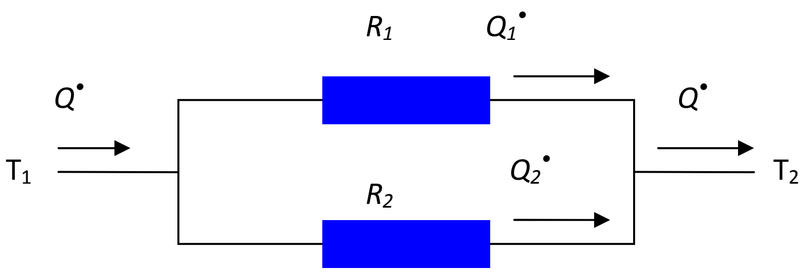


Fig. 3: Parallel resistances.

$$Q^{\bullet} = Q_1^{\bullet} + Q_2^{\bullet} = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$Q^{\bullet} = \frac{T_1 - T_2}{R_{total}}$$

$$\frac{1}{R_{total}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \rightarrow \frac{1}{R_{total}} = \frac{R_1 R_2}{R_1 + R_2}$$

Example 1: Thermal Resistance Network

Consider the combined series-parallel arrangement shown in figure below. Assuming one –dimensional heat transfer, determine the rate of heat transfer.

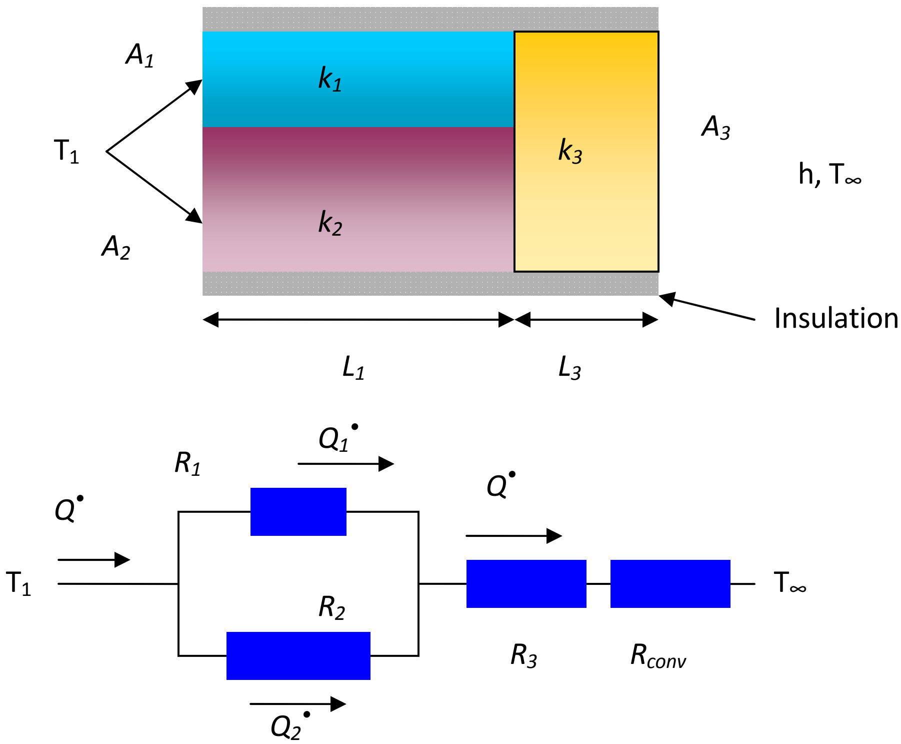


Fig. 4: Schematic for example 1.

Solution:

The rate of heat transfer through this composite system can be expressed as:

$$Q^{\bullet} = \frac{T_1 - T_\infty}{R_{total}}$$

$$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$

Two approximations commonly used in solving complex multi-dimensional heat transfer problems by transfer problems by treating them as one dimensional, using the thermal resistance network:

1- Assume any plane wall normal to the x-axis to be isothermal, i.e. temperature to vary in one direction only T = T(x)

2- Assume any plane parallel to the x-axis to be adiabatic, i.e. heat transfer occurs in the x-direction only.

These two assumptions result in different networks (different results). The actual result lies between these two results.

Heat Conduction in Cylinders and Spheres

Steady state heat transfer through pipes is in the normal direction to the wall surface (no significant heat transfer occurs in other directions). Therefore, the heat transfer can be modeled as steady-state and one-dimensional, and the temperature of the pipe will depend only on the radial direction, T = T (r).

Since, there is no heat generation in the layer and thermal conductivity is constant, the Fourier law becomes:

$$\dot{Q}_{cond,cyl} = -kA\frac{dT}{dr} \qquad (W)$$

$$A = 2\pi rL$$

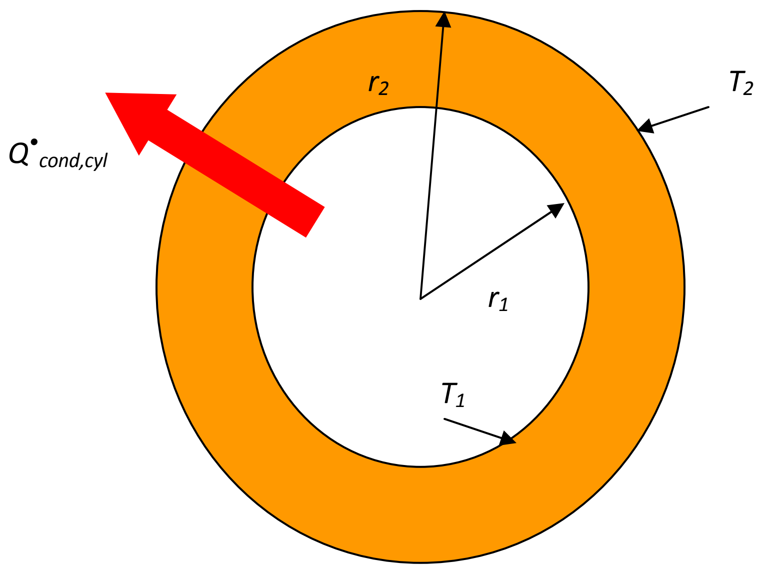


Fig. 5: Steady, one-dimensional heat conduction in a cylindrical layer.

After integration:

$$\int_{r1}^{r2} \frac{\dot{Q}_{cond,cyl}}{A} dr = -\int_{T1}^{T2} k dT \qquad A = 2\pi rL$$

$$\dot{Q}_{cond,cyl} = 2\pi kL \frac{T_1 - T_2}{\ln(r_2 / r_1)}$$

$$\dot{Q}_{cond,cyl} = \frac{T_1 - T_2}{R_{cyl}}$$

$$R_{cyl} = \frac{\ln(r_2 / r_1)}{2\pi kL}$$

where R_{cyl} is the conduction resistance of the cylinder layer.

Following the analysis above, the conduction resistance for the spherical layer can be found:

$$\dot{Q}_{cond,sph} = \frac{T_1 - T_2}{R_{sph}}$$

$$R_{sph} = \frac{r_2 - r_1}{4\pi \, r_1 r_2 k}$$

The convection resistance remains the same in both cylindrical and spherical coordinates, $R_{conv} = 1/hA$. However, note that the surface area $A = 2\pi rL$ (cylindrical) and $A = 4\pi r^2$ (spherical) are functions of radius.

Example 2: Multilayer cylindrical thermal resistance network

Steam at $T_{\infty,1}$ = 320 °C flows in a cast iron pipe [k = 80 W/ m.°C] whose inner and outer diameter are D_1 = 5 cm and D_2 = 5.5 cm, respectively. The pipe is covered with a 3-cm-thick glass wool insulation [k = 0.05 W/ m.°C]. Heat is lost to the surroundings at $T_{\infty,2}$ = 5°C by natural convection and radiation, with a combined heat transfer coefficient of h_2 = 18 W/m^2. °C. Taking the heat transfer coefficient inside the pipe to be h_1 = 60 W/m^{2}K, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drop across the pipe shell and the insulation.

Assumptions:

Steady-state and one-dimensional heat transfer.

Solution:

Taking L = 1 m, the areas of the surfaces exposed to convection are:

$A_1 = 2\pi r_1 L = 0.157\ m^2$

$A_2 = 2\pi r_2 L = 0.361\ m^2$

$$R_{conv,1} = \frac{1}{h_1 A_1} = \frac{1}{\left(60\, W/m^2.^\circ C\right)\left(0.157 m^2\right)} = 0.106\ ^\circ C/W$$

$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = 0.0002^\circ C/W$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = 2.35^\circ C/W$$

$$R_{conv,2} = \frac{1}{h_2 A_2} = 0.154\ ^\circ C/W$$

$$R_{total} = R_{conv,1} + R_1 + R_2 + R_{conv,2} = 2.61^\circ C/W$$

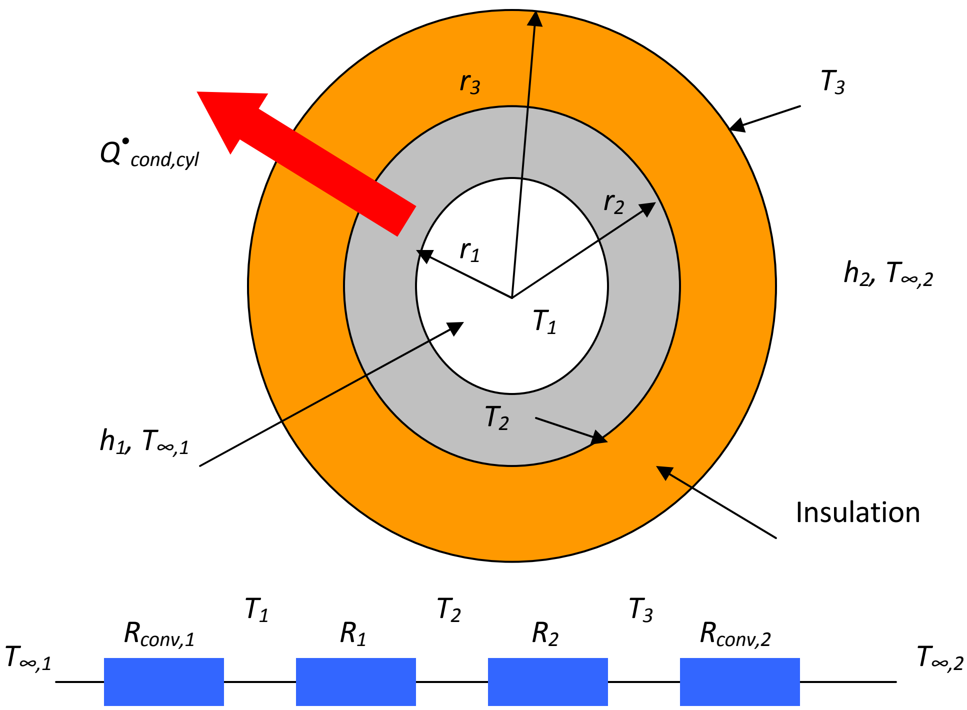


Fig. 6: Schematic for example 1.

The steady-state rate of heat loss from the steam becomes

$$Q^{\bullet} = \frac{T_{\infty,1} - T_{\infty,2}}{R_{total}} = 120.7\ W \qquad \text{(per m pipe length)}$$

The total heat loss for a given length can be determined by multiplying the above quantity by the pipe length.

The temperature drop across the pipe and the insulation are:

$$\Delta \mathrm{T}_{\mathrm{pipe}} = Q^{\bullet} R_{pipe} = (120.7\ W)(0.0002\ {}^{\circ}C/W) = 0.02^{\circ}C$$
$$\Delta \mathrm{T}_{\mathrm{insulation}} = Q^{\bullet} R_{insulation} = (120.7\ W)(2.35\ {}^{\circ}C/W) = 284^{\circ}C$$

Note that the temperature difference (thermal resistance) across the pipe is too small relative to other resistances and can be ignored.

Critical Radius of Insulation

To insulate a plane wall, the thicker the insulator, the lower the heat transfer rate (since the area is constant). However, for cylindrical pipes or spherical shells, adding insulation results in increasing the surface area which in turns results in increasing the convection heat transfer. As a result of these two competing trends the heat transfer may increase or decrease.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi kL} + \frac{1}{(2\pi r_2 L)h}}$$

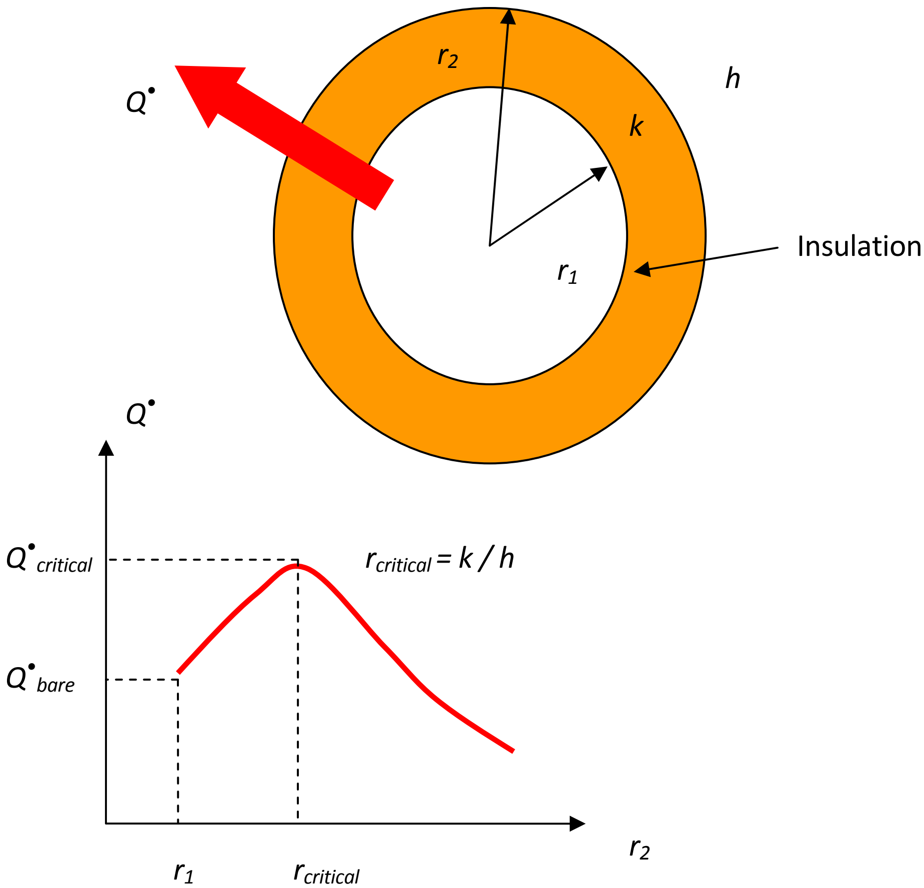


Fig. 7: Critical radius of insulation.

The variation of $\dot{Q}$ with the outer radius of the insulation reaches a maximum that can be determined from $d\dot{Q} / dr_2 = 0$. The value of the critical radius for the cylindrical pipes and spherical shells are:

$$r_{cr,cylinder} = \frac{k}{h} \qquad (m)$$

$$r_{cr,spherer} = \frac{2k}{h} \qquad (m)$$

Note that for most applications, the critical radius is so small. Thus, we can insulate hot water or steam pipes without worrying about the possibility of increasing the heat transfer by insulating the pipe.